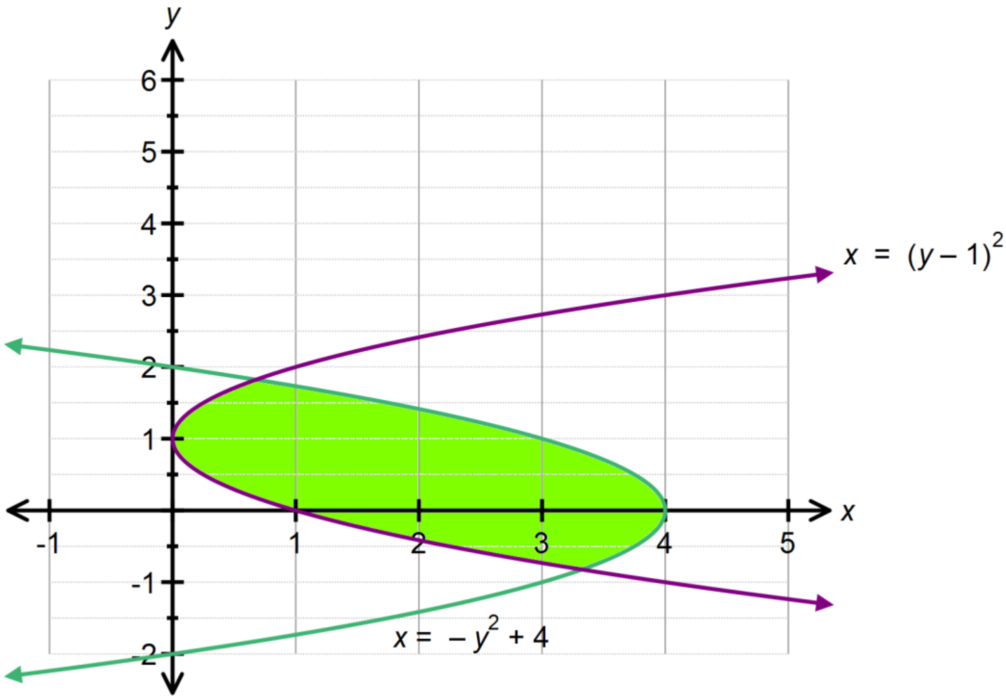
Calculator Assumed
Applications of Anti-Differentiation 2

Time: 45 minutes
Total Marks: 45
Your Score: / 45

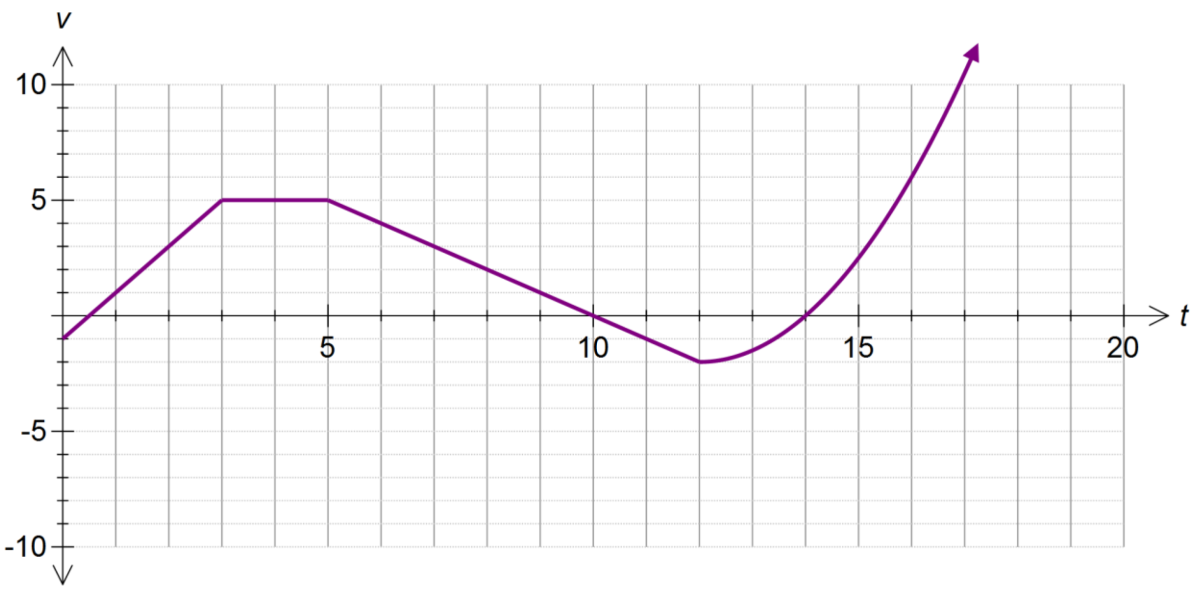
Question One: [6 marks] **CA**

Calculate the shaded area shown below, showing all relevant working.



Question Two: [1, 1, 2, 3, 2, 3, 2, 5 = 19 marks] CA

A particle moving in rectilinear motion has its velocity function graphed below, where t is time in seconds and v is in ms^{-1}.



(a) Determine the initial speed of the particle.

(b) Determine the acceleration of the particle during the 4^{th} second.

(c) Calculate the displacement of the particle after 3 seconds.

(d) Calculate the distance travelled by the particle in the first 12 seconds.

(e) Determine when the particle has travelled a distance of 21 m since commencement.

(f) State the times when the particle was at rest.

(g) When did the particle first return to the origin?

(h) Calculate the distance travelled by the particle for $13 \le t \le 18$ if it is known that the velocity for $t \ge 12$ is given by $v(t) = at^2 + bt + c$.

Question Three: [6, 3 = 9 marks] CA

Sybil has invested $\$A$ in a fund which compounds her investment continuously at a rate of k% per annum.

The rate of change of her investment is given by $\frac{dV}{dt} = k(Ae^{kt})$ where V is the value of her investment in dollars and t is the time in years.

The net change in the value of her investment in the first 10 years is \$12 331 . 78.

The net change in the value of her investment in the next 10 years is \$22 469 . 97.

(a) Determine the values of A and k.

(b) Hence determine the function that defines the value of her investment.

Question Four: [2, 2, 3, 4 = 11] CA

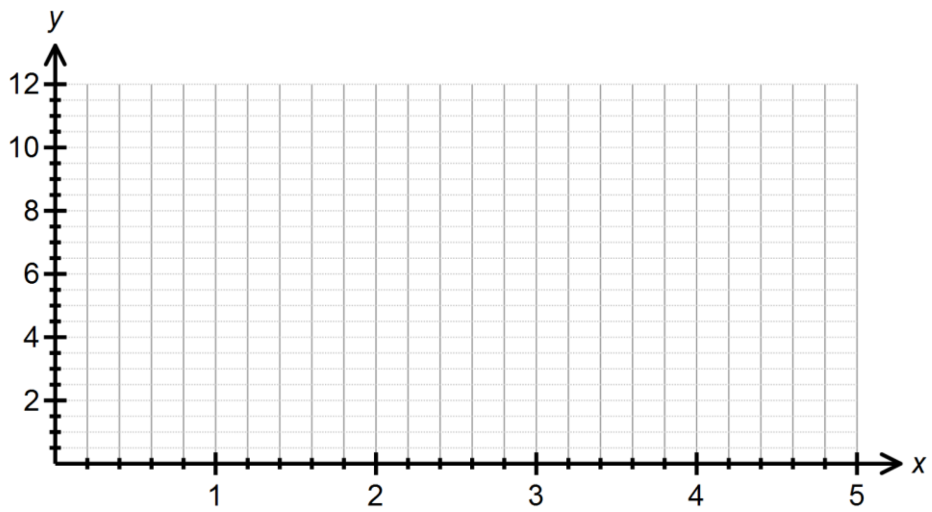
We can use integration to determine the arc length of a curve. The development of this idea is similar to developing the idea of the area under a curve as the sum of infinitely many rectangles, but instead is built on the use of Pythagoras' Theorem for smaller and smaller right triangles.

The arc length of a section of curve, $a \leq x \leq b$ is given by:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Consider the function $f(x) = 2x + 1$

(a) Graph this function over the domain $0 \leq x \leq 5$ on the graph below.



(b) Use Pythagoras' Theorem to determine the length of the line drawn above.

(c) Use the arc length formula provided above to confirm that you obtain the same answer as in part (b).

Consider the function $g(x) = e^{2x}\sin(2x)$.

(d) Calculate the length of the curve of $g(x)$ over the domain $1 \le x \le 2$

SOLUTIONS
Calculator Assumed
Applications of Anti-Differentiation 2

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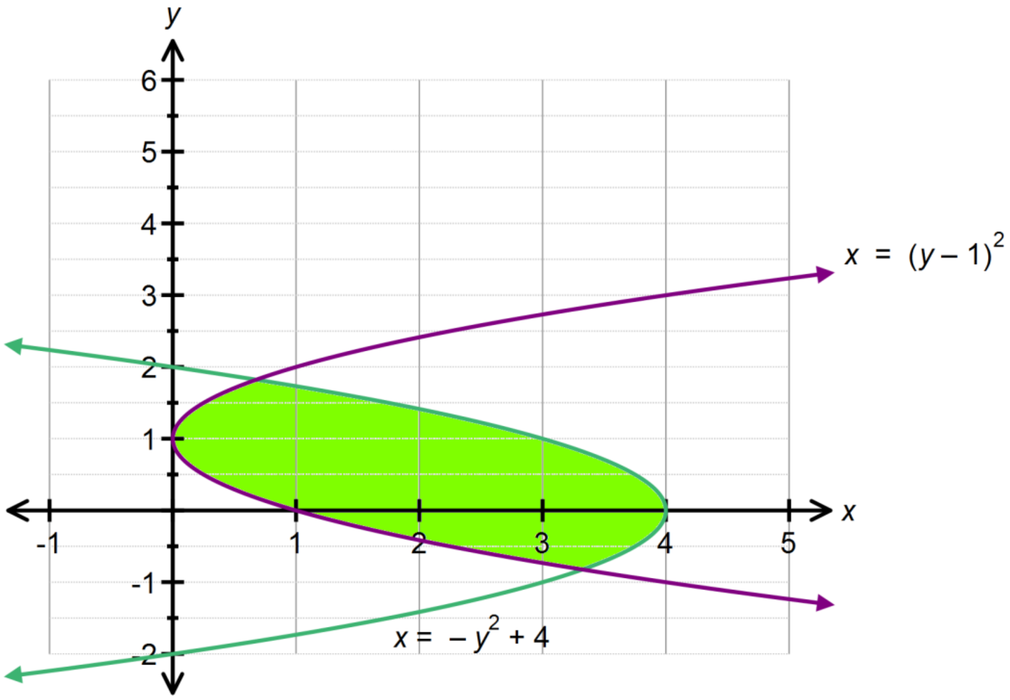
Question One: [6 marks] **CA**

Calculate the shaded area shown below, showing all relevant working.



$(y-1)^2 = -y^2 + 4$ ✓

$y = 1.83, \ \ y = -0.83$ ✓

$x = 0.68, \ \ x = 3.32$

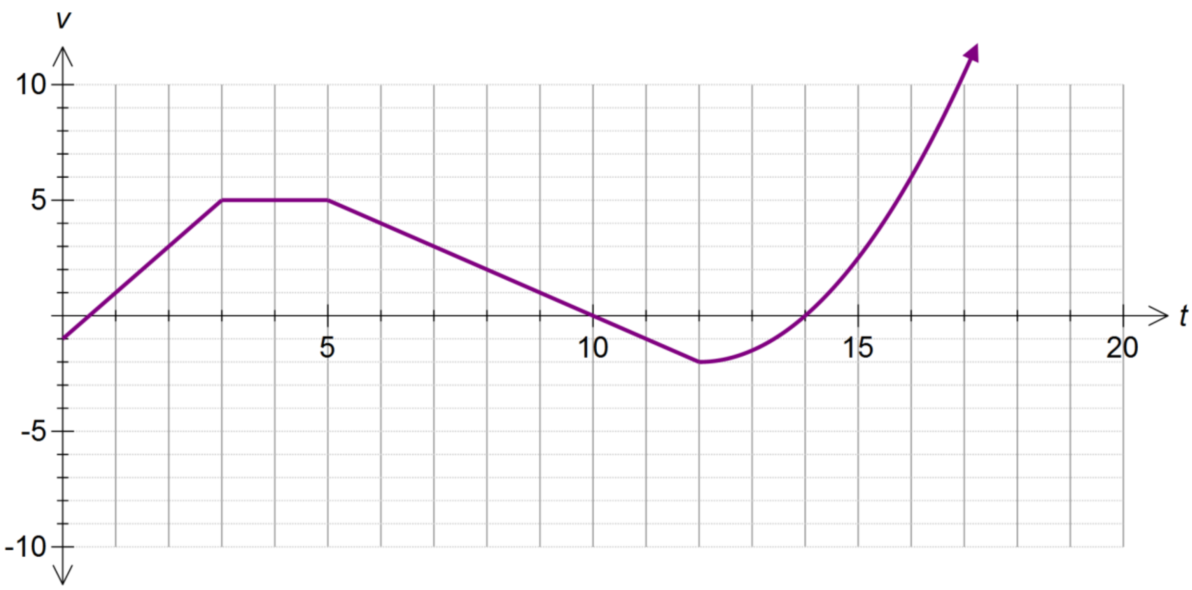
$Area = \int_{-0.83}^{1.83} \left[(-y^2+4)-(y-1)^2\right] dy$ ✓

$= \left[\frac{-y^3}{3} + 4y - \frac{(y-1)^3}{3}\right]_{-0.83}^{1.83}$ ✓✓

$= 6.173 units^2$ ✓

Question Two: [1, 1, 2, 3, 2, 3, 2, 5 = 19 marks] CA

A particle moving in rectilinear motion has its velocity function graphed below, where *t* is time in seconds and *v* is in ms⁻¹.



(a) Determine the initial speed of the particle.

$|v(0)| = 1m/s$ ✓

(b) Determine the acceleration of the particle during the 4th second.

Slope of the line between t =3 and t =4 is 0. Therefore $a(t) = 0\ m/s^2$ ✓

(c) Calculate the displacement of the particle after 3 seconds.

$x(3) = -(0.5\times0.5\times1) + (0.5\times2.5\times5)$ ✓
$x(3) = 6m$ ✓

(d) Calculate the distance travelled by the particle in the first 12 seconds.

$= (0.5\times0.5\times1) + 0.5\times5(2+9.5) + (0.5\times2\times2)$ ✓ ✓
$= 31m$ ✓

(e) Determine when the particle has travelled a distance of 21 m since commencement.

Distance in first 5 seconds: 16.5m ✓

Distance in the 6th second: 4.5 m

Therefore 6 seconds. ✓

(f) State the times when the particle was at rest.

$t = 0.5,\ 10,\ 14$ ✓ ✓ ✓

(g) When did the particle first return to the origin?

$x(t) = 0$ ✓

$t = 1s$ ✓

(h) Calculate the distance travelled by the particle for $13 \le t \le 18$ if it is known that the velocity for $t \ge 12$ is given by $v(t) = at^2 + bt + c$.

$pts: (12, -2)\ (14, 0)\ (16, 6)$ ✓

$\therefore v(t) = 0.5t^2 - 12t + 70\ (via\ regression)$ ✓✓

$dist = \int_{13}^{18} |v(t)|\,dt$ ✓

$dist = 27.5m$ ✓

Question Three: [6, 3 = 9 marks] **CA**

Sybil has invested $\$A$ in a fund which compounds her investment continuously at a rate of $k\%$ per annum.

The rate of change of her investment is given by $\frac{dV}{dt} = k(Ae^{kt})$ where V is the value of her investment in dollars and t is the time in years.

The net change in the value of her investment in the first 10 years is \$12 331 . 78.

The net change in the value of her investment in the next 10 years is \$22 469 . 97.

(a) Determine the values of A and k.

$$\int_0^{10} kAe^{kt}\ dt = 12331.78$$ ✓

$$\int_{10}^{20} kAe^{kt}\ dt = 22469.97$$ ✓

$$\left[Ae^{kt}\right]_0^{10} = 12331.78$$

$$Ae^{10k} - A = 12331.78 \quad (1)$$ ✓

$$\left[Ae^{kt}\right]_{10}^{20} = 22469.97$$

$$Ae^{20k} - Ae^{10k} = 22469.97 \quad (2)$$ ✓

$$k = 0.06 \quad A = 15000$$ ✓ ✓

(b) Hence determine the function that defines the value of her investment.

$$V(t) = 15000\,e^{0.06t}$$ ✓ ✓ ✓

Question Four: [2, 2, 3, 4 = 11] CA

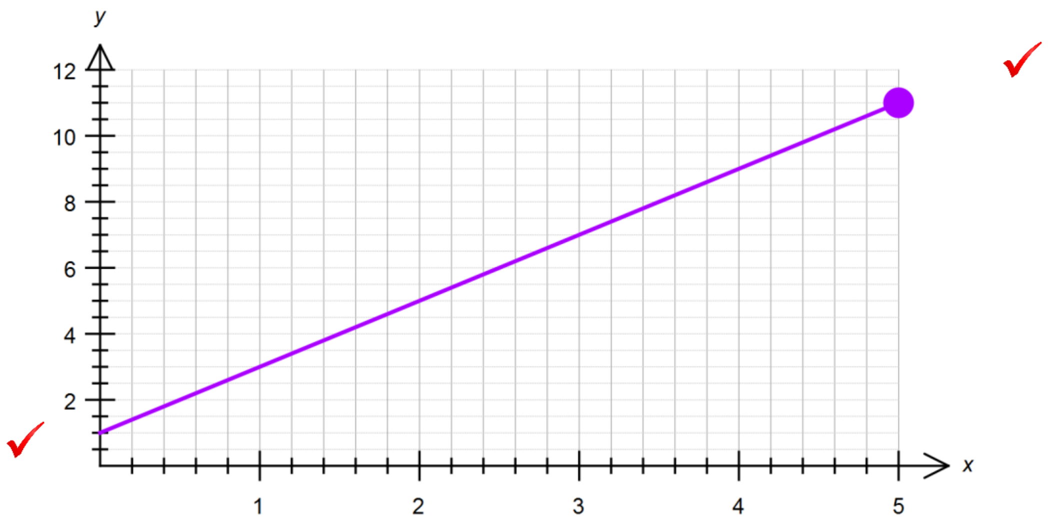
We can use integration to determine the arc length of a curve. The development of this idea is similar to developing the idea of the area under a curve as the sum of infinitely many rectangles, but instead is built on the use of Pythagoras' Theorem for smaller and smaller right triangles.

The arc length of a section of curve, $a \leq x \leq b$ is given by:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Consider the function $f(x) = 2x + 1$

(a) Graph this function over the domain $0 \leq x \leq 5$ on the graph below.



(b) Use Pythagoras' Theorem to determine the length of the line drawn above.

$length = \sqrt{5^2 + 10^2} = 11.18\,units$

(c) Use the arc length formula provided above to confirm that you obtain the same answer as in part (b).

$L = \int_0^5 \sqrt{1 + (2)^2}\, dx$

$L = \left[\sqrt{5}x\right]_0^5$

$L = 5\sqrt{5} = 11.18$

Consider the function $g(x) = e^{2x}\sin(2x)$.

(d) Calculate the length of the curve of $g(x)$ over the domain $1 \le x \le 2$

$$L = \int_1^2 \sqrt{1 + (2e^{2x}\sin(2x) + 2e^{2x}\cos(2x))^2}\,dx$$

$$L = 49.59 \text{ units}$$